



# ArcGIS™ Survey Analyst concepts

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## An ESRI Technical Paper

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# ArcGIS™ Survey Analyst concepts

## **Introduction**

Surveying is the science of collecting measurements to determine the relative spatial locations of points on or near the surface of the earth. Relative spatial locations are represented by coordinates. Coordinates stored in a GIS are used to represent physical objects that depict natural and man-made features on a map.

To establish coordinates for points, surveyors use precise field instruments, procedures, and computations. They measure slope, horizontal and vertical distances between points, and angles between lines of sight.

Measurements, computations, survey points, and coordinates, collectively called survey objects, are stored in a survey dataset in the geodatabase.

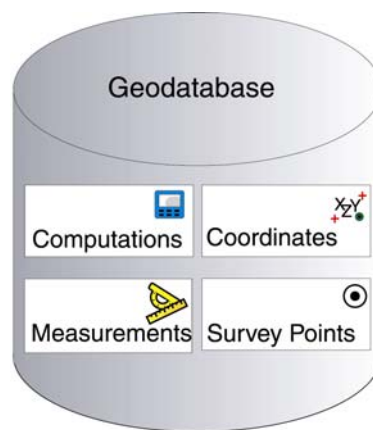
In addition to storing these objects, Survey Analyst can track dependencies between computations. Computations define points that can be used as input for other computations. These dependencies are modeled in Survey Analyst as a computation network.

Physical objects are represented in a geodatabase as features stored in feature classes. Survey Analyst enhances feature classes with survey-awareness, allowing stored features to be associated with survey data.

This technical paper explains the survey data model, describes the computation network, and presents the concept for associating GIS features with survey data.

## Survey data in the geodatabase

Like other geographic information, survey measurement data can be managed in a GIS using database management system (DBMS) tables. It is possible to store this information as an integral part of the geodatabase. This technical paper presents the key concepts and an overview of the survey data model based on the four data types: measurements, survey points, coordinates, and computations. The survey data model includes four data types that are used to analyze and solve problems related to the processing of survey data stored in the geodatabase:



A survey dataset contains four object classes:

- Survey points: named locations that are observed through various surveys. Survey points can be observed multiple times and by many surveys over time. One goal is to improve the location of survey points with new survey measurements.

- Coordinates: survey points can have many coordinates associated with their location, especially as new surveys are performed through time. The location of each survey point is improved and becomes more accurate with each new survey. A survey point can have multiple coordinates, but there is always one coordinate that is used for publication to the GIS layers or used in computations. GIS feature geometry can be linked to the location of the survey point. Thus, feature geometry can be improved over time as the survey point's coordinates are more accurately located.

- Measurements

- Computations

Collectively, the computations, measurements, coordinates, and survey points are called survey objects, and are stored in tables called survey classes.

Coordinates for the named survey points are calculated through a series of survey measurements and computations. This forms the core of the survey information collected and computed using Survey Analyst.

Survey Analyst is a system for surveyors to use field surveys and other sources of survey measurement information to calculate and update coordinates of survey points.

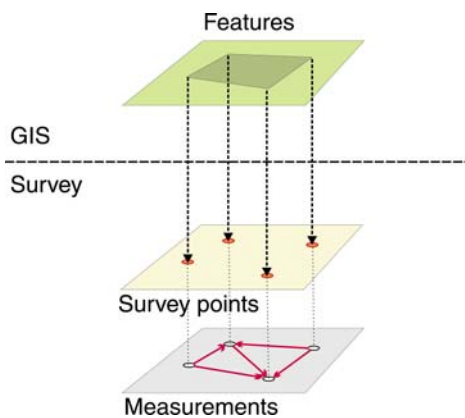
The four data types—survey points, coordinates, measurements, and computations—are managed in one comprehensive dataset. Each new survey adds records to this dataset and is used to update the GIS coordinate for each survey point. A new survey can result in new survey points being added as well.

Each new survey is managed as a survey project and is used to incorporate new survey information in the single comprehensive survey dataset.

A key part of the Survey Analyst user interface is the toolset used to create new survey projects, import the new survey data into the survey dataset, perform computations, and update the survey point locations.

In addition to comprehensive survey data management, the other key goal of Survey Analyst is to incrementally improve the accuracy of GIS feature geometry in the geodatabase as the survey accuracy is improved. This is done by linking features to survey point locations. During the initial design stage, you declare that selected feature classes are survey-aware. Feature geometry in survey-aware feature classes can be linked and optionally adjusted to move feature coordinates to survey point locations. And, over time, as the survey point locations are improved, feature geometry can be adjusted as well.

Survey measurements are used to compute survey points. Survey points are used to update the geometry of survey-aware features:



## Performing a survey

A typical work flow for a surveyor starts with gathering preliminary information. The first phase of a survey often includes activities such as the following:

- Performing an initial reconnaissance survey in the field
- Identifying existing physical control points
- Finding coordinates for these control points based on an official source
- Deciding on a coordinate system
- Deciding on the equipment and methods that should be used based on the required coordinate accuracy for the survey

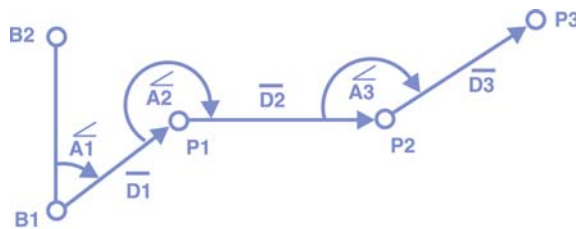
The second phase of a survey focuses on the collection of the measurements and other information from the field. This data is captured electronically or on paper and usually comprises sketches and field books.

The third phase requires calculating coordinates for measured locations by entering and processing them in well-known computations, such as the traverse. The sequence of steps performed to collect the data in the field usually determines the set of computations that are used to calculate the coordinates.

The coordinates computed in surveys may be used for the following purposes:

- Further numerical analysis
- In the case of cadastral surveys, further legal boundary analysis
- Subsequent work in the field
- Creation of plans for submission to a government authority or a private client

The following graphic demonstrates the third phase of a survey project, which requires calculating coordinates by processing measurements collected in the field:



**ImportPoint** [ ( X1,Y1 ) → B1 ]

**ImportPoint** [ ( X2,Y2 ) → B2 ]

**Computation1** [ Traverse (Course1 ( B1,  $\overleftarrow{A1}$ , B2,  $\overline{D1}$ )) → P1  
 (Course2 ( P1,  $\overleftarrow{A2}$ , B1,  $\overline{D2}$ )) → P2  
 (Course3 ( P2,  $\overleftarrow{A3}$ , P1,  $\overline{D3}$ )) → P3 ]

## The survey project



The survey data model includes the survey project. The survey project represents a unit of work, and is used as a logical structure that owns and manages a group of measurements, points, coordinates, and computations that function and belong together.

The survey project is the geodatabase equivalent of the electronic and paper artifacts collected and generated through the three survey project phases described in the previous section. Survey projects are used for managing survey workflow as part of a comprehensive survey database.


As subsequent field surveys collect more measurements from the field, additional survey projects are created to enter this new information into the geodatabase.

When creating a survey project, you give it a name and define its coordinate system. Once added to the geodatabase, a survey project is a candidate for owning any new survey data added to the geodatabase.

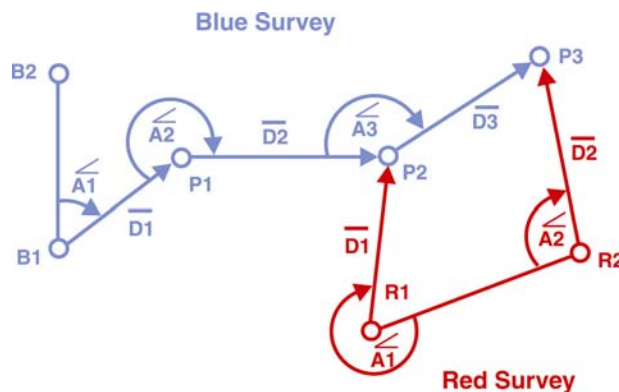
The information collected for each survey is grouped into separate units of work called projects:

Measurements 		Coordinates 		
FromTo	Type	PointName	Type	Coordinate
B1-B2	Reference Direction	B1	Imported	xyz
B1-P1	Angle	B2	Imported	xyz
B1-P1	Distance	P1	Computed	xyz
P1-B1	Reference Direction	P2	Computed	xyz
P1-P2	Angle	P3	Computed	xyz
P1-P2	Distance			
P2-P1	Reference Direction			
P2-P3	Angle			
P2-P3	Distance			

Computations 	
Name	Type
Computation1	Traverse

In the following example, the data collected for the Red survey is added to a survey project called Red:





Through time, surveyors can collect discrete sets of survey data that can be accumulated into a single dataset:

Survey Projects		
ID	Name	Coordinate System
1	Blue	NAD_1927_UTM_Zone_12N
2	Red	NAD_1927_StatePlane_Arizona_Central

**ImportPoint [ ( X1,Y1 ) → R1 ]**

**ImportPoint [ ( X2,Y2 ) → R2 ]**

**Computation1 [AngleDistance (R1,  $\overset{\angle}{A1}$ , R2,  $\overset{-}{D1}$ ) → P3**

**Computation2 [AngleDistance (R2,  $\overset{\angle}{A2}$ , R1,  $\overset{-}{D2}$ ) → P2**

Records added as a result of new survey activity are referenced in the geodatabase as a survey project:

Measurements		Coordinates		
FromTo	Type	PointName	Type	Coordinate
B1-B2	Reference Direction	B1	Imported	xyz
B1-P1	Angle	B2	Imported	xyz
B1-P1	Distance	P1	Computed	xyz
P1-B1	Reference Direction	P2	Computed	xyz
P1-P2	Angle	P3	Computed	xyz
P1-P2	Distance	R1	Imported	xyz
P2-P1	Reference Direction	R2	Imported	xyz
P2-P3	Angle	P2	Computed	xyz
P2-P3	Distance	P3	Computed	xyz
R1-R2	Reference Direction			
R1-P2	Angle			
R1-P2	Distance			
R2-R1	Reference Direction			
R2-P3	Angle			
R2-P3	Distance			

Computations	
Name	Type
Computation1	Traverse
Computation1	AngleDistance
Computation2	AngleDistance

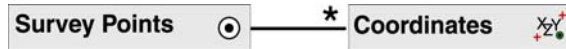
### Modeling survey points and coordinates

Surveyors identify physical locations in the field by naming the points that they measure. In the survey data model, named locations are known as survey points.

In different surveys, the same physical locations can be computed with different coordinates. For instance, the named location P2 in this example has one set of coordinates measured in the Blue survey and another set measured in the Red survey. This is also the case for the point called P3.

Survey points represent multiple coordinates, and are modeled as separate tables in the database.

There is a one-to-many relationship between survey points and coordinates:



### The GIS coordinate

You can define a single coordinate for a survey point that is the best representation for its location. This coordinate is called the GIS Coordinate. Any number of survey projects can contribute to this value. It may be the coordinate from a single survey project, or it may be the weighted average from a number of different projects.

Survey Points			Coordinates		
Point Name	GIS Coordinate	ID	Survey Point ID	Type	Coordinate
B1	XYZ	101	101	Imported	xyz
B2	XYZ	102	102	Imported	xyz
P1	XYZ	103	103	Computed	xyz
P2	XYZ	104	104	Computed	xyz
P3	XYZ	105	105	Computed	xyz
R1	XYZ	106	106	Imported	xyz
R2	XYZ	107	107	Imported	xyz
			104	Computed	xyz
			105	Computed	xyz

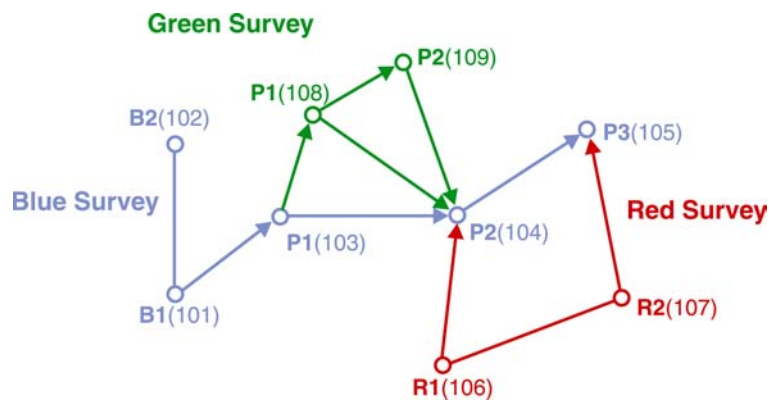
Weighted mean from Red and Blue projects

**The current coordinate**

Just as there is a GIS coordinate to represent the best overall coordinate for a survey point from many projects, there is also a best coordinate representation within each project, called the current coordinate. A current coordinate is required when the same project computes or imports more than one coordinate for a particular survey point. In this example, the Green project has two computed coordinates for point P2:

Coordinates						Survey Points			
ID	Project ID	Survey Point ID	Type	Coordinate	Current	Project ID	Point Name	GIS Coordinate	ID
1	1	101	Imported	xyz	yes	1	B1	XYZ	101
2	1	102	Imported	xyz	yes	1	B2	XYZ	102
3	1	103	Computed	xyz	yes	1	P1	XYZ	103
4	1	104	Computed	xyz	yes	1	P2	XYZ	104
5	1	105	Computed	xyz	yes	1	P3	XYZ	105
6	2	106	Imported	xyz	yes	1	R1	XYZ	106
7	2	107	Imported	xyz	yes	1	R2	XYZ	107
8	2	104	Computed	xyz	yes	3	P1	XYZ	108
9	2	105	Computed	xyz	yes	3	P2	XYZ	109
10	3	103	Imported	xyz	yes				
11	3	108	Computed	xyz	yes				
12	3	104	Computed	xyz	yes				
13	3	109	Computed	xyz	yes				
14	3	104	Computed	xyz	no				

————— Current coordinate  
————— Noncurrent coordinate

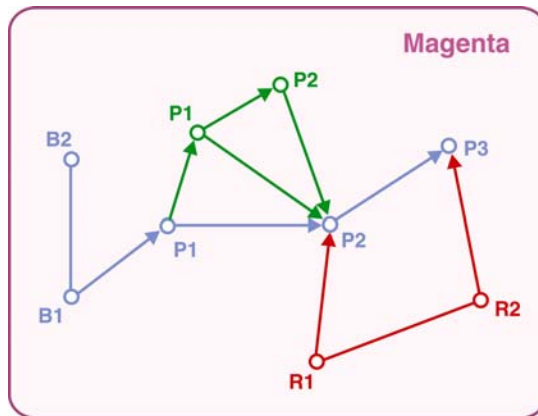


When defining the GIS coordinate, only the current coordinate of each survey project is used in the weighted average. Within the project, the current coordinate is used for any computations that require the survey point as input. In this example, Blue P2's GIS coordinate is the weighted mean of coordinates with ObjectIDs 4, 8, and 12.

**Working with multiple projects**

Although measurements and survey points can be owned only by a single survey project, their database records can be shared with other projects. This means that you can create a survey project that uses existing measurements and survey points to create records for computations and coordinates.

The Magenta project does not own new measurements and survey points. It uses the existing survey points and measurements stored in the survey dataset:




Survey Projects		
ID	Name	Coordinate System
1	Blue	NAD_1927_UTM_Zone_12N
2	Red	NAD_1927_StatePlane_Arizona_Central
3	Green	NAD_1927_StatePlane_Arizona_Central
4	Magenta	NAD_1927_UTM_Zone_12N


In the following example, the Magenta project makes use of measurements and points from the other three projects to define a single new least squares adjustment computation and new coordinates for a predefined survey point in the Blue project:

**Computation1 [ Least Squares Adjustment**  
**Input :** ( P1,R1,R2,B1,P1,P2,  $\overrightarrow{P1-P1}$ ,  $\overrightarrow{P1-P2}$ ,  $\overrightarrow{P1-P2}$ ,  $\overrightarrow{P1-P2}$ ,  $\overrightarrow{P2-P2}$ ,  $\overrightarrow{R1-P2}$  )  
**Output :** ( P2 ) ]


The Magenta project copies coordinates from survey points and computes a new coordinate for P2 of the Blue project:

Coordinates 					
ID	Project ID	Survey Point ID	Type	Coordinate	Current
1	1	101	Imported	xyz	yes
2	1	102	Imported	xyz	yes
3	1	103	Computed	xyz	yes
4	1	104	Computed	xyz	yes
5	1	105	Computed	xyz	yes
6	2	106	Imported	xyz	yes
7	2	107	Imported	xyz	yes
8	2	104	Computed	xyz	yes
9	2	105	Computed	xyz	yes
10	3	103	Imported	xyz	yes
11	3	108	Computed	xyz	yes
12	3	104	Computed	xyz	yes
13	3	109	Computed	xyz	yes
14	3	104	Computed	xyz	no
15	4	103	Copied	xyz	yes
16	4	106	Copied	xyz	yes
17	4	107	Copied	xyz	yes
18	4	101	Copied	xyz	yes
19	4	108	Copied	xyz	yes
20	4	109	Copied	xyz	yes
21	4	104	Computed	xyz	yes

Survey Points 			
Project ID	Point Name	GIS Coordinate	ID
1	B1	XYZ	101
1	B2	XYZ	102
1	P1	XYZ	103
1	P2	XYZ	104
1	P3	XYZ	105
1	R1	XYZ	106
1	R2	XYZ	107
3	P1	XYZ	108
3	P2	XYZ	109

Computations 		
Project ID	Name	Type
1	Computation1	Traverse
2	Computation1	AngleDistance
2	Computation2	AngleDistance
3	Computation1	Traverse
4	Computation1	Least Squares

**The survey dataset** Through time, surveyors collect discrete sets of survey data that can be accumulated into a single dataset—a survey dataset. The survey dataset is a comprehensive collection of survey information in the geodatabase. It consists of tables that store records for the four survey data types (survey classes). The following is a basic description of the core tables of a survey dataset:

Computations		
Project ID	Name	Type
1	Computation1	Traverse
2	Computation1	AngleDistance
2	Computation2	AngleDistance

Measurements		
Project ID	FromTo	Type
1	B1-B2	Reference Direction
1	B1-P1	Angle
1	B1-P1	Distance
1	P1-B1	Reference Direction
1	P1-P2	Angle
1	P1-P2	Distance
1	P2-P1	Reference Direction
1	P2-P3	Angle
1	P2-P3	Distance
2	R1-R2	Reference Direction
2	R1-P2	Angle
2	R1-P2	Distance
2	R2-R1	Reference Direction
2	R2-P3	Angle
2	R2-P3	Distance

Survey Points			
Project ID	Point Name	GIS Coordinate	ID
1	B1	XYZ	101
1	B2	XYZ	102
1	P1	XYZ	103
1	P2	XYZ	104
1	P3	XYZ	105
2	R1	XYZ	106
2	R2	XYZ	107

Coordinates			
Project ID	Survey Point ID	Type	Coordinate
1	101 (B1)	Imported	xyz
1	102 (B2)	Imported	xyz
1	103 (P1)	Computed	xyz
1	104 (P2)	Computed	xyz
1	105 (P3)	Computed	xyz
2	106 (R1)	Imported	xyz
2	107 (R2)	Imported	xyz
2	104 (P2)	Computed	xyz
2	105 (P3)	Computed	xyz

Survey Projects		
ID	Name	Coordinate System
1	Blue	NAD_1927_UTM_Zone_12N
2	Red	NAD_1927_StatePlane_Arizona_Central

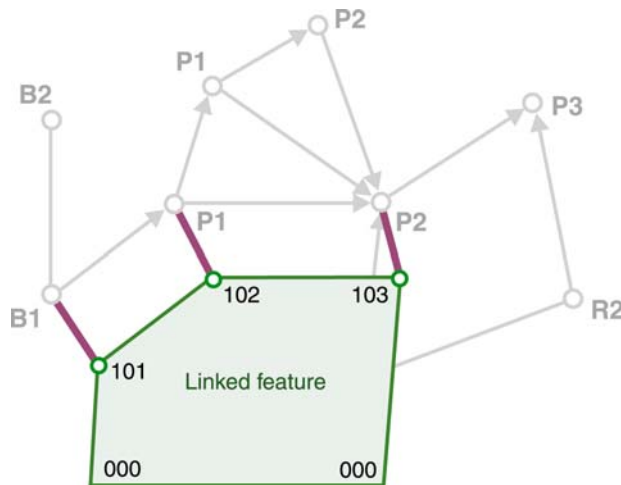
**Survey-aware feature classes**

One key benefit of having a survey dataset as part of your geodatabase is that you can associate the geometry of features with the stored survey data.

Features that are associated with survey data are called survey-aware features. Similarly, the feature classes in the geodatabase that contain these features are called survey-aware feature classes.

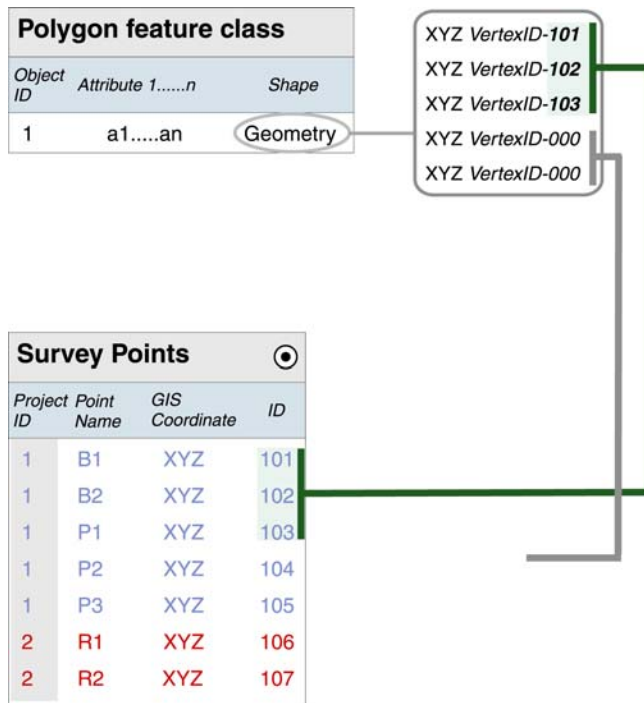
The survey points stored in the survey dataset provide a framework of coordinate control for the geometry of survey-aware features. You can make links between each survey point's GIS coordinate and the vertex of one or more features. A linked feature vertex does not need to share the same location as the survey point.

Survey points can be linked to survey-aware features. In the following graphic, the line between B1 and 101 represents a link, points 103 and 000 represent Vertex IDs of the feature, and the survey point is represented by the point P2 (where all arrows are pointing):

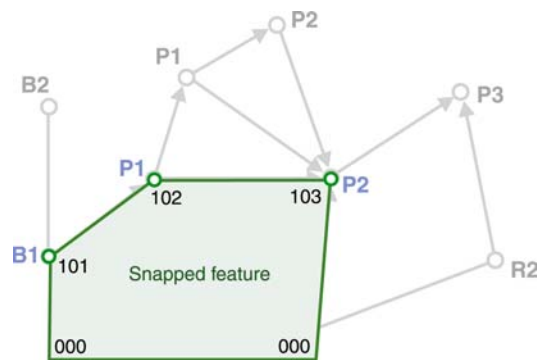




Links are maintained in the geometry attributes of features. Each linked vertex stores a VertexID number that matches the unique identifier of its linked survey point. Vertex IDs are positive integers; 0 means no link.



Using links, features can be snapped to survey points:



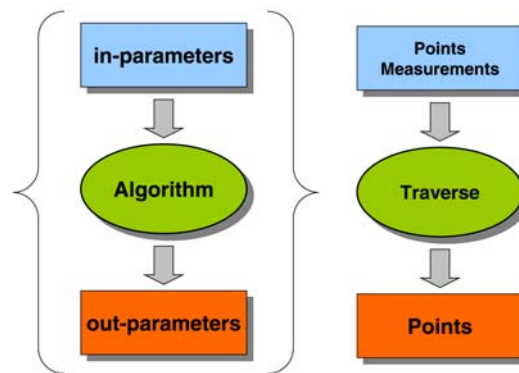


## Modeling computations

A computation is a process that requires a predefined set of input parameters to apply a set of rules and an algorithm to calculate output parameters.

For a specific kind of computation, the type and number of input parameters are predefined. The computed coordinates can be added either to new survey points or to existing survey points. Computations store references to existing survey points and measurements, and use these measurement values and coordinates to calculate new coordinates. After a computation has been processed successfully for the first time, new coordinates are stored and referenced as the output of the computation.

Computations store references to survey points and measurements. These are used as input parameters to calculate new coordinates for survey points:



## Computation states

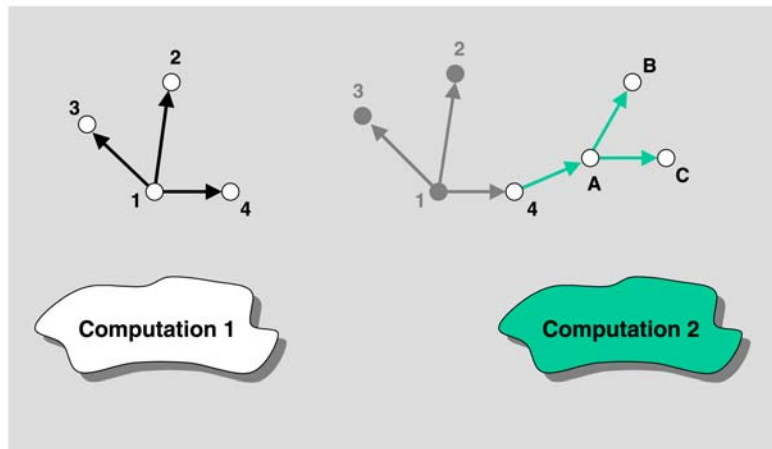
If a computation's input measurements or points are altered, the computation and its output coordinates will be out-of-date. The computation must be recomputed to become valid. Recomputing also updates the output coordinates.

Out-of-date and valid are two states for computations. They can also be *incorrect* when predefined limits are exceeded, or *incomplete* when they do not have all the required input.

**Computation dependencies**

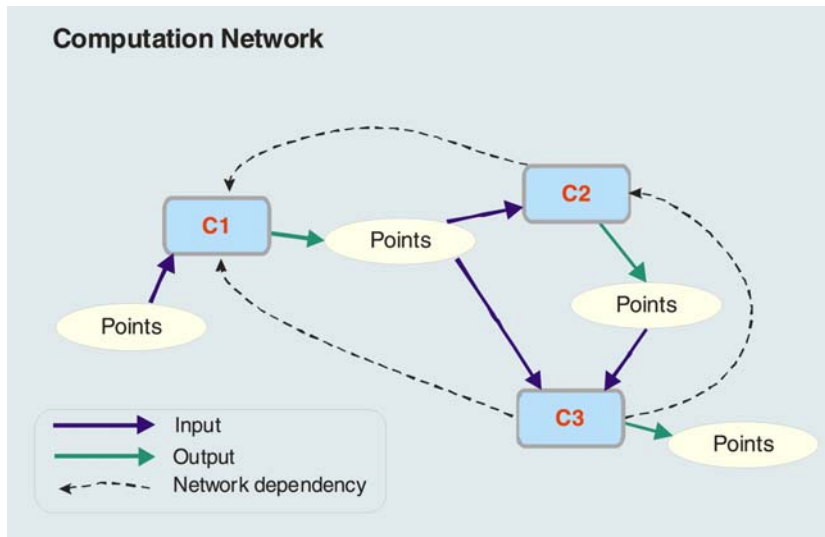
A dependency between two computations occurs when one computation uses the results of another. Survey point coordinates are often calculated in a sequence—the output coordinates of computations are used as input for others. This creates computation dependencies.

In the following graphic, Computation 2 is dependent on point 4. Since point 4 is defined by Computation 1, Computation 2 depends on Computation 1.



**Computation networks** A computation network is a sequence of computation dependencies—the output points of some computations are used as the inputs for one or more others.

A computation network models this sequence of computations, by tracking dependencies and managing the associated changes in computation states. It maps dependencies between computations in a survey project:



Using the functions of Survey Analyst, you can rerun individual computations or entire networks. When reprocessing a network, the system detects all the source computations that are out-of-date and updates downstream computations, setting their states to valid. During this process, some dependent computations may be assigned incorrect states. In these cases, the relevant downstream computations are not processed and remain in the out-of-date state.

**Multiple computation types** Since there are many different kinds of computations with a wide variety of formats for their required inputs and outputs, each type of computation has its own table in the survey dataset.

**Modeling measurements**

Survey Analyst defines two primary surveying systems for modeling measurements. The first models coordinate geometry, and the second models measurements from theodolite-based field equipment (TPS). All measurements can be modeled in two categories: simple measurements and composite measurements. This section describes these measurement categories for both coordinate geometry and TPS surveying systems.

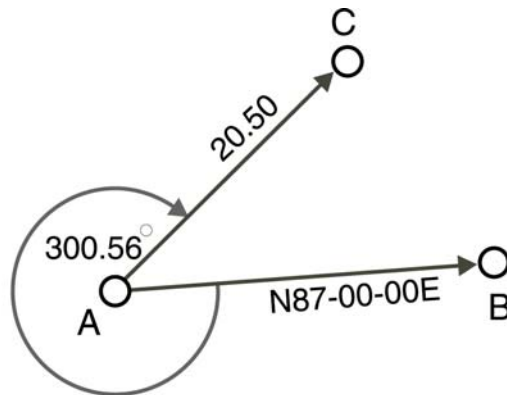
**Modeling COGO measurements**

Values that define vectors, directions, lengths, and orthogonal offsets are modeled using COGO simple measurements.

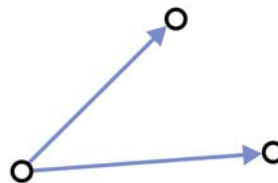
A COGO composite measurement is used to represent measurements that are dependent on others. For example, a deflection angle requires a direction for the angle value to be meaningful.

A COGO composite measurement depends on COGO simple measurements for its definition.

The following example shows how a composite measurement requires a simple measurement to represent a deflection angle:



COGO Simple measurement						
Project ID	From point	To point	Type	Direction	Distance	ID
01	A	B	Direction	N87E	-	201
01	A	C	Distance	-	20.50	202



COGO Composite measurement						
Project ID	From point	To point	Type	Value	Simple Meas ID	ID
01	A	C	Angle	300.56	201	501



**Modeling field measurements**

Another example of a simple measurement is an entry in a field book that represents observations from a theodolite: a slope distance, vertical angle, horizontal angle, and a height of target. This type of simple measurement is called a TPS measurement.

Field measurements from a theodolite are modeled as a field book with pages of recorded observations. Each line entry is a TPS measurement, and each set of entries represents a single instance of an instrument setup.

TPS Setup				
Point Name : X		Date: 02-04-2002		
Height of instrument: 4.52ft		Time: 11:20 am		
Temperature: -				
Pressure: -				
To Point	Horizontal Angle	Vertical Angle	Slope Distance	Target Height
A	0° 00' 02"			
B	41° 21' 26"	91° 21' 26"	55.63	5.00
C	115° 38' 52"	90° 55' 20"	113.65	5.00
D	203° 56' 55"	88° 58' 09"	205.69	5.00
E	332° 44' 20"	90° 10' 40"	198.94	5.00
A	0° 00' 05"			

A composite measurement is a group of simple measurements that are related and applied as a group. In the case of field measurements, a group of field book entries that belong together define a single setup of the instrument. This type of composite measurement is called a TPS setup. A TPS setup is comprised of a set of TPS measurements.

Each observation (slope distance, vertical angle, horizontal angle, and height of target) is recorded as a TPS measurement and is added to the TPS setup.

**Modeling survey object dependencies**

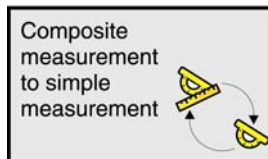
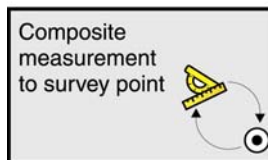
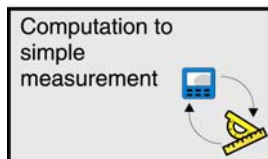
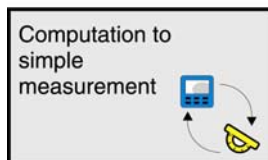
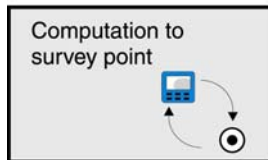
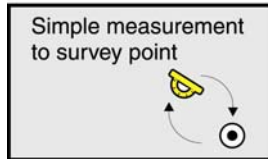
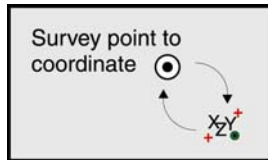
As discussed in this chapter, a relationship between survey points and coordinates exists. There are also other relationships between measurements, computations, coordinates, and survey points. These relationships define dependencies between survey objects. It is necessary to be aware of these dependencies when working in the Survey Analyst environment.

The following list of dependencies are enforced and maintained by the survey dataset as a set of relationship tables:

- A survey point can contain multiple coordinates.
- Many computations can define coordinates for the same survey point.
- Many survey points can be created or used by one computation.
- Many measurements can start and end at the same two survey points.
- The same measurement value can be used between many pairs of survey points.
- Many computations can use the same measurement.
- Many measurements can be used in a single computation.
- Multiple simple measurements are used within a composite measurement.
- Many composite measurements can use the same simple measurement.

These relationship tables in the survey dataset manage and enforce the dependencies between survey object classes.

The graphic on the next page shows relationship tables for survey object classes:



**Least squares adjustment—overview****What are measurements?**

The techniques and algorithms supported by the least squares computation provides the most rigorous method available for processing the observations in a survey network. This section provides a review of the basics of these techniques.

A measurement is an observed numerical value that is an estimate of the “true” size of a quantity.

Measurements are best described by comparing measuring with counting. Put simply, counting determines exact numbers and measuring does not. For example, if several people are each asked to count the number of fish in a tank, their counted values are either right or wrong. There is no uncertainty in the result because the exact number of fish in the tank is a realistic, known target.

By contrast, consider the following scenario: several people are each, in turn, given a tape measure and asked to calculate the volume of water in the fish tank by measuring the length and breadth of the tank, and the depth of the water. They are asked to do this as accurately as possible and to estimate fractions of the measurement unit of the tape measure.

Each person will derive a different volume. Which of these is correct? Unlike the exact quantity of fish, there is no exact quantity for the volume of water with which to compare results.

Most calculated volumes for the tank are likely to be close to the “true” value, while some others may be incorrect due to mistakes in one or more of the tank’s measurements. Uncertainty in measurements is an accepted truth. It exists because:

- The observer makes estimates.
- There is imperfection in the measuring equipment.
- The environment in which equipment and observer operate affects the measurement.
- The behavior of equipment, observer, and the environment cannot always be predicted.

These last four points can be categorized into two groups: measurement error, and mistakes. If you measure with care, mistakes can be avoided. However, the same measurements will always contain error. As illustrated in the fish tank example, a measurement with a mistake is not useful for calculating the volume of water. It should be removed, and the quantity remeasured. However, a measurement with error is expected. You expect the different measurers to obtain different values for the volume of water.

Surveyors recognize and work within this environment of mistakes and errors. To get close to the “true” value of the measured quantity, it is important to identify and remove mistakes, and to apply mathematical and statistical methods to deal with measurement error.



**Measurement error**      Measurement error is classified as either systematic or random.

Systematic error follows a mathematical or physical law and can be corrected to comply with a known standard. Corrections are always applied the same way—for example, if a tape measures a standard 100-foot baseline as 99.89 feet, then every measurement of this magnitude has a systematic error of 0.11 feet. A correction of 0.11 feet is always added to the measured value. Surveyors calibrate measurement equipment to reduce systematic error.

Random error is arbitrary; it follows the laws of statistics and probability—the size and sign of the error cannot be predicted. Random error is handled within the context of the following assumptions:

- A plus sign error will occur as frequently as a minus sign error.
- Small errors will occur more frequently than large errors.
- The chance for large errors to occur is small.

When there are a large number of repeated measurements of a specific quantity, a pattern of random error distribution emerges. After the mistakes are removed and the systematic errors accounted for, the statistical model is based on a theoretical normal probability distribution. For more information, see the ‘Stochastic Model’ section later in this technical paper.

Coordinate quality—  
precision versus  
accuracy

Surveyors are required to assess and control the quality of their work. One level of quality assessment for surveyors is based on their published coordinates. Since measurements are used to define coordinates, measurement error gets propagated into the calculated coordinates.

Assessment of quality is based on the precision and accuracy of measured values.

Precision is the closeness to one another of a repeated set of observations of the same quantity. It is a measure of the control over random error.

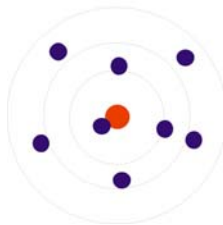
Accuracy can be defined as closeness to a theoretical “truth”.

A frequently used example that distinguishes accuracy from precision is the grouping of darts on a target. Like repeated observations of the same quantity, these dart throws can be:

- Precise and accurate
- Precise but inaccurate
- Accurate but imprecise

Using the bulls-eye as the true value, you can visualize the center of gravity for each of these groupings.

The following graphic demonstrates precision and accuracy. The first grouping of dart throws is both precise and accurate, the second grouping is precise but inaccurate, and the third grouping is imprecise but accurate because its center of gravity falls close to the bulls-eye.



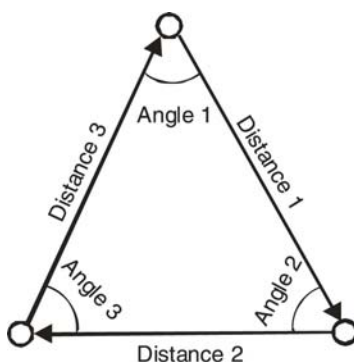
It is apparent that though the second grouping is precise, accuracy without precision is closer to the truth than precision without accuracy. Naturally, both precision and accuracy are desirable. Precision and random error are directly related. When the size of random errors decreases, the precision increases. Similarly, accuracy and systematic error are directly related. When systematic errors can be correctly predicted, the result is an increase in accuracy.

## Redundancy in measurements

It becomes evident that uncertainty caused by random error is reduced by repeating measurements of the same quantity. However, this is not enough to detect systematic error. For instance, repeated taped distances between the same two building corners will not detect that the tape measure has not been correctly calibrated.

It is common practice for a surveyor to establish a network of measurements in which each survey point has been measured from a number of other survey points. This results in improved quality in the final coordinates. The more redundancy you have in a measurement network, the better your chances will be of detecting and controlling problems. Redundancy occurs when the number of measurements is greater than the number of parameters to be calculated. A simple example of this is the set of measurements between the three points of a triangle, as depicted in the figure below.

The triangle below has redundant information. All of the measured angles and distances are useful for describing the triangle's geometry, but they do not define a unique solution. Where redundancy like this exists, a least squares adjustment solves for a "best-fit" solution.



Since there are extra measurements, a unique solution that exactly fits the triangle is not possible. For example, the angles of a triangle should add up to exactly 180 degrees, but due to error as described in the previous sections, this will not occur. The least squares adjustment is the ideal tool for solving for an optimal solution where redundancy exists in your measurement network.

### Least squares adjustment

Since redundancy exists in measurement networks, a method is needed to correct the measurements to make them fit the conditions as well as possible. In the example of the preceding section, the conditions are defined by the geometry of a triangle. The amount by which each measurement must be corrected is called the measurement residual.

The least squares adjustment method defines a best-fit solution by finding a minimum for the sum of the squares of the measurement residuals.

The final measurement residuals are called the least squares corrections.

Least squares adjustment models consist of two important components: the mathematical model and the stochastic model. The mathematical model is a set of relations between the measurements and the unknown coordinates. The stochastic model describes the expected error distribution of the measurements.

### Mathematical model

In your surveys, the measurements are often not the final quantities that you require. Measurements are processed in computations to define coordinates for survey points. Through computations, coordinates are expressed as a function of the measurements. Each computation, therefore, defines a mathematical model. In the case of the least squares adjustment, the mathematical model forms a basis for the least squares adjustment.

A least squares adjustment requires the location, orientation, and scale of the measurement network to be defined.

In case of a network where measurements are reduced to two dimensions, there are two translations, one rotation, and one scale factor. In this case, at least two reference points are required (two Eastings and two Northings). This places a minimum set of four constraints on the solution for defining location, orientation, and scale.

### Stochastic model

The stochastic model of a least squares adjustment describes the statistical (stochastic) deviations of measurements. The variation in measurements of a single quantity, as described in the preceding sections, is modeled by assuming a normal probability distribution. This distribution is based on the mean  $\mu$  and the standard deviation  $\sigma$  of a measured quantity.

The mean  $\mu$  is a mathematical representation for the best expected value of the measured quantity. The standard deviation  $\sigma$  is a measure of the dispersion or spread of the probability, and characterizes the precision of the measurement. The square of  $\sigma$  is called the variance.

By definition, there is a 0.684 probability that normally distributed stochastic variables will fall within a window limited by  $-\sigma$  and  $+\sigma$ . For a window limited by  $-2\sigma$  and  $+2\sigma$ , this probability is 0.954.

It is possible for two or more measurements to be correlated. This means that a deviation in one measurement will influence the other. This correlation is reflected in the computed coordinates.

The correlation between coordinates x, y, and z is mathematically expressed in a 3x3 matrix, called a variance–covariance matrix.

$$\begin{pmatrix} Q_{xx} & & \\ Q_{xy} & Q_{yy} & \\ Q_{xz} & Q_{yz} & Q_{zz} \end{pmatrix}$$

In the data model for the survey datasets, the variance–covariance matrix is used to express the probability distribution for survey point coordinates and provides a quantitative estimate of survey point quality. Since the matrix is symmetrical, the values of the variance–covariance matrix can be expressed as six attribute values in the tables for the survey points and coordinates.

For each measurement, a standard deviation  $\sigma$  is chosen. The value for  $\sigma$  is based on knowledge about the measurement process (conditions in the field and type of instrument) and experience.

The precision of the coordinates computed in the adjustment depends on the precision of the measurements and on the propagation of this precision through the mathematical model.

The least squares adjustment formulae

The (linearized) mathematical model is expressed as follows:

$$\underline{y} = AX + \underline{e} + a$$

with

$$\underline{y} = (m) \text{ vector of observations;}$$

$$\underline{e} = (m) \text{ vector of corrections;}$$

$$A = (m \times n) \text{ design matrix;}$$

$$x = (n) \text{ vector of unknowns;}$$

$$a = (m) \text{ vector of constants.}$$

The stochastic model is:

$$Q_y = \sigma^2 Q = \underline{1} P^{-1}$$

with

$$Q_y = (m \times m) \text{ variance-covariance matrix;}$$

$$\sigma^2 = \text{a-priori variance-of-unit-weight;}$$

$$Q = (m \times m) \text{ weight coefficient matrix;}$$

$$P = (m \times m) \text{ weight matrix.}$$

The least squares criterion is:

$$\underline{e}^t P \underline{e} = \text{minimum}$$

The solution is:

$$\underline{x} = (A^t P A)^{-1} A^t P (\underline{y} - a)$$

$$s^2 = \frac{\underline{e}^t P \underline{e}}{m-n}$$

with

$$(A^t P A) = N = (n \times n) \text{ normal matrix;}$$

$$s^2 = \text{a-posteriori variance-of-unit-weight.}$$

The variance-covariance matrix of the unknown values is given as:

$$Q_x = \sigma^2 N^{-1}$$

As shown in the preceding formulae, the least squares approach requires a set of linear equations.

The solution for the vector of unknown  $\underline{x}$  is available after a series of iterative updates  $d\underline{x}$  of the approximate values  $x^0$ :

$$\underline{x} = x^0 + d\underline{x}$$

After each iteration, the new solution is compared with the previous one. If the difference between the two solutions is negligible, the iteration process converges and is ended with the final values defined by the results of the last iteration.

**Survey points, measurements, and the spatial reference**

The geodatabase stores information about the geometry or shape of an object as a field in a table. This shape field represents a geometry type (point, line, polygon, or multipoint) and a sequential set of x and y coordinates that optionally contain z and m values and vertex IDs.

Geometry stored in a shape field requires a spatial reference to relate it to the surface of the earth. The spatial reference has two components: a coordinate system and a spatial domain.

The coordinate system is used to project coordinates from a mathematical approximation of the earth's surface, called the ellipsoid, to the planar surface of a map.

The spatial domain is the minimum and maximum value for the geometry attributes. The extent of this domain defines the precision at which geometry attributes (x, y, z, m, ID) can be stored as integers. There are a finite number of integers available in the system, so the x and y spatial domain is analogous to a square grid that always contains the same number of rows and columns (approximately two billion, or 2 to the power of 31).

Precision and the spatial extent are inversely proportional. Since the x and y values must correspond to the intersection of lines in this theoretical grid, the larger the chosen extent, the lower the precision of the geometry attributes.

The spatial reference for features in the geodatabase is stored as a property of a feature dataset or standalone feature class. Similarly, the spatial reference for survey points and measurements is stored as a property of a survey dataset. Each survey project also has its own coordinate system but does not require a spatial domain. This is described in the next section.

For more information about spatial references, see the ESRI Press book *Understanding Map Projections*.

**Maintaining numerical precision**

When processing survey data in computations, it is important to maintain numerical precision. Since the coordinates stored in the shape field are snapped to an integer grid, they do not provide sufficient precision for this purpose. Therefore, the survey data model enforces the maintenance of x,y,z coordinates in separate fields of double precision. Only these fields are used when performing computations. (Measurement values are also stored in fields of double precision.)

Additionally, the survey projects in the survey dataset each have independent coordinate systems.

An update of a feature based on links to survey points results in the feature vertices being snapped to the geometry of the survey point.

**Summary**

This technical paper presents a number of different concepts that are helpful to understand before you start using the software. For more information on using Survey Analyst, see the book *Using ArcGIS™ Survey Analyst*.



